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Some Properties of the Matched, Symmetrical Six-Port Junction

GORDON P. RIBLET AND E. R. BERTIL HANSSON

Abstract —**Based on the S-matrix element-eigenvalue relations, the basic features of the matched, symmetrical, reciprocal six-port junction are derived. It is shown to be unsuitable for use in a six-port measurement system but can be used to build a five-way power divider. The equivalent admittance of the junction is derived, and, as an application, a stripline five-way power divider is designed. The theory is confirmed by the close agreement between computed and measured performance of an experimental one-to-five power divider.**

I. Introduction

ONE CLASS OF devices with, in general, n-fold axial symmetry are the symmetrical junctions. It is noteworthy that reciprocal junctions of this type up to and including the five-fold symmetrical junction were treated already in 1948 by Dicke [1]. Among the large number of other works on symmetrical junctions should be mentioned the book *Nonreciprocal Microwave Junctions and Circulators* by Helszajn [2]. However, as far as is known, the reciprocal, symmetrical six-port junction has never been given a detailed study in the literature.

Through the last few years, considerable interest has been focused on the six-port measurement technique, due largely to an important series of papers by Engen and Hoer [3]–[6]. The types of six-ports proposed so far have mostly been relatively complex, as in [6]. Attempts made to find simple types of suitable six-ports have resulted in narrow-band devices [7], [8] with one exception, namely the symmetrical, reciprocal five-port junction combined with a directional coupler [9], [10]. One object of this paper is to examine the suitability of the symmetrical, reciprocal six-port junction for making six-port measurements.

II. Basic Properties of the Matched, Symmetrical, Reciprocal Six-Port Junction

A symmetrical six-port junction can be described by six complex quantities. Choosing initially for our description the scattering matrix, we have, at most, six different entries. By diagonalizing the scattering matrix, we get an alternative description of the junction in terms of the six eigenvalues of the scattering matrix. These eigenvalues constitute the reflection coefficients of the six possible eigenexcitations of the junction. The principle of conservation of energy requires that the scattering matrix eigenvalues be of unit amplitude for a lossless junction. A simple set of relations exists between the scattering matrix elements and its eigenvalues

$$S_{1I} = \frac{1}{6} \sum_{J=1}^{6} S_J e^{j(I-1)(J-1)(\pi/3)}, \qquad I = 1,2,\cdots,6 \quad (1)$$

where S_{1I} are the elements and S_I are the eigenvalues of the scattering matrix.

For a reciprocal, symmetrical six-port junction $S_{15} = S_{13}$ and $S_{16} = S_{12}$. From (1) it follows that $S_5 = S_3$ and $S_6 = S_2$.

Manuscript received April 4, 1983; revised September 6, 1983.

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Thus, the element-eigenvalue relations for the reciprocal, symmetrical six-port junction are

$$S_{11}=\frac{1}{6}(S_1+2S_2+2S_3+S_4) \tag{2}$$

$$S_{12}=\frac{1}{6}(S_1+S_2-S_3-S_4) \tag{3}$$

$$S_{13}=\frac{1}{6}(S_1-S_2-S_3+S_4) \tag{4}$$

$$S_{14}=\frac{1}{6}(S_1-2S_2+2S_3-S_4). \tag{5}$$

Some interesting conclusions about the properties of symmetrical, reciprocal six-port junctions can be drawn directly from (2)–(5). In particular, we will treat the case of a matched junction, i.e., $S_{11}=0$.

One way of matching the junction would be to choose $S_4=-S_1$ and $S_3=-S_2$. Insertion in (2)–(5) gives

$$S_{11}=0 \tag{6}$$

$$S_{12}=\frac{1}{3}(S_1+S_2) \tag{7}$$

$$S_{13}=0 \tag{8}$$

$$S_{14}=\frac{1}{3}(S_1-2S_2). \tag{9}$$

Here, we notice that S_{13} is identically zero, while the relative amplitudes of S_{12} and S_{14} depend upon S_1 and S_2. With $S_2=-S_1$ we get for instance $S_{11}=S_{12}=S_{13}=0$ and $S_{14}=S_1$. Such a circuit constitutes a matched three-signal-crossover without interference between signals. Another choice of interest is $S_2=S_1\cdot e^{j\pi/3}$, which gives $S_{11}=S_{13}=0$, $S_{12}=S_1\cdot e^{j\pi/6}/\sqrt{3}$, and $S_{14}=S_1\cdot e^{-j\pi/2}/\sqrt{3}$. Here, an input signal at port 1 is divided equally between ports 2, 4, and 6 with a phase difference of 120° between S_{12} and S_{14}. A similar result is obtained for $S_2=S_1\cdot e^{-j\pi/3}$. Assuming the junction to be lossless, that is $|S_1|=|S_2|=1$, it is impossible to choose S_1 and S_2 so that $|S_{14}|=0$ as shown by (9). The minimum value of $|S_{14}|$ occurs for $S_2=S_1$ in which case $S_{12}=2S_1/3$ and $S_{14}=-S_1/3$.

The choice of $S_4=-S_1$ and $S_3=-S_2$ treated above is not the only way to match the symmetrical, reciprocal six-port junction. Another possibility is illustrated in Fig. 1, which gives a geometrical interpretation of (2) with $S_{11}=0$. The junction is here assumed lossless so that $|S_1|=|S_2|=|S_3|=|S_4|=1$. Further, $\arg(S_1)$ is taken equal to zero. Different solutions are generated by varying the arguments of S_2, S_3, and S_4 so that the point a follows the external circle and point b the internal circle. An additional set of solutions is obtained by reflexion of the previous ones in the real axis (see the dashed-line example in Fig. 1). By changing the relative angles of the scattering matrix eigenvalues, we can manipulate the signal amplitudes of all the three transmission variables S_{12}, S_{13}, and S_{14}. If, for instance, we take $S_2=S_1\cdot e^{j\pi/2}$, $S_3=-S_1\cdot e^{j\lambda}$ and $S_4=S_1\cdot e^{-j(\pi/2-\lambda)}$, where $\lambda=\sin^{-1}(0.6)$ we find from (2)–(5) that $S_{11}=0$, $|S_{12}|=|S_{13}|=|S_{14}|=1/\sqrt{5}$, $\arg(S_{13})=\arg(S_{12})-\pi/2$, $\arg(S_{14})=\arg(S_{12})-\pi$. The solution of (2) for the case of a matched five-way equal power divider is illustrated in Fig. 2. The dual solution obtained by reflexion in the real axis (see Fig. 2) gives the same amplitude distribution but changes the sign of the offset angles so that $\arg(S_{13})=\arg(S_{12})+\pi/2$. Thus, the possibility of constructing a matched one-to-five equal power divider by the use of a symmetrical, reciprocal six-port junction has been demonstrated.

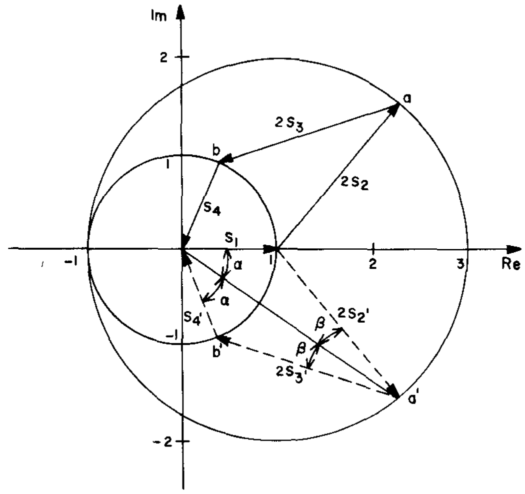

Fig. 1. Geometrical illustration of the matching condition for a lossless, symmetrical, reciprocal six-port junction.

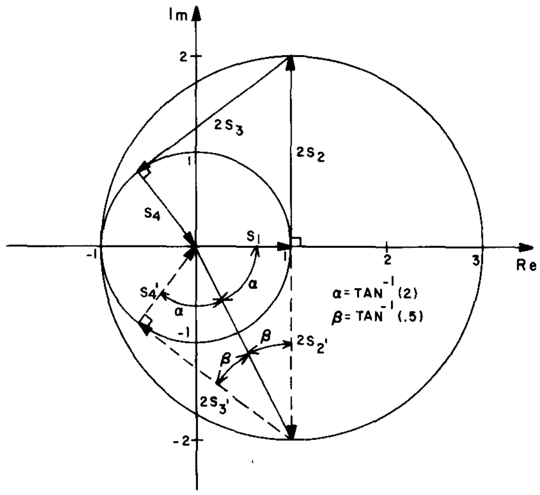

Fig. 2. Illustration of the matching condition for a lossless, symmetrical, reciprocal six-port junction as a five-way equal power divider.

From the various examples given above, it is obvious that matching a symmetrical, reciprocal six-port junction is not enough to ensure a unique signal distribution.[1] This fact tends to increase the design problems and to make any circuits narrowband.

[1]Compare with the symmetrical, nonreciprocal three-port junction and the symmetrical, reciprocal five-port junction where a specific coupling is a consequence of matching alone [9].

III. The Matched, Symmetrical, Reciprocal Six-Port Junction in Six-Port Measurements

In order to be useful for the accurate determination of complex signal ratios, a six-port should possess certain properties. Engen showed that, ideally, one power detector connected to the six-port should measure the power incident to the unknown load. In addition, the complex numbers q_1, q_2, and q_3 associated with the three other power detectors should be symmetrically distributed around the origin in the complex plane [5] so that $|q_1|=|q_2|=|q_3|$, $\arg(q_2)=\arg(q_1)\pm 120°$, and $\arg(q_3)=\arg(q_1)\mp 120°$.

Fig. 3 shows a symmetrical, reciprocal six-port junction in a configuration for six-port measurements. The waves entering into and emerging from the six-port junction, assuming matched detectors so that $a_2=a_4=a_5=a_6=0$, are related by

$$b_2 = S_{12}a_1 + S_{12}a_3 \tag{10}$$

$$b_3 = S_{13}a_1 \tag{11}$$

$$b_4 = S_{14}a_1 + S_{12}a_3 \tag{12}$$

$$b_5 = S_{13}a_1 + S_{13}a_3 \tag{13}$$

$$b_6 = S_{12}a_1 + S_{14}a_3. \tag{14}$$

Observing that $|S_{13}| \neq 0$ is required for a signal to be incident on the test device, we eliminate a_1 by the aid of (11)

$$b_2 = \frac{S_{12}}{S_{13}} b_3 + S_{12}a_3 \tag{15}$$

$$b_4 = \frac{S_{14}}{S_{13}} b_3 + S_{12}a_3 \tag{16}$$

$$b_5 = b_3 + S_{13}a_3 \tag{17}$$

$$b_6 = \frac{S_{12}}{S_{13}} b_3 + S_{14}a_3. \tag{18}$$

A requirement on the six-port is that one of the power detectors should measure the power incident on the test device. Since $|S_{13}| \neq 0$, there are two possible ways to accomplish this by choosing either S_{12} or S_{14} equal to zero.

1) $S_{12}=0$:

$$P_2 \sim |b_2|^2 = 0 \tag{19}$$

$$P_4 \sim |b_4|^2 = \left|\frac{S_{14}}{S_{13}}\right|^2 |b_3|^2 \tag{20}$$

$$P_5 \sim |b_5|^2 = |S_{13}|^2|b_3|^2\left|\Gamma - \left(-\frac{1}{S_{13}}\right)\right|^2 \tag{21}$$

$$P_6 \sim |b_6|^2 = |S_{14}|^2|b_3|^2|\Gamma - 0|^2 \tag{22}$$

where $\Gamma = a_3/b_3$. In (21) and (22), we identify the q-values as $-1/S_{13}$ and 0. No power emerges to detector 1.

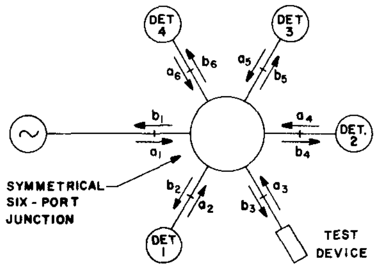

Fig. 3. The symmetrical six-port junction in a simple configuration for six-port measurements.

2) $S_{14}=0$:

$$P_2 \sim |S_{12}|^2|b_3|^2\left|\Gamma - \left(-\frac{1}{S_{13}}\right)\right|^2 \tag{23}$$

$$P_4 \sim |S_{12}|^2|b_3|^2|\Gamma - 0|^2 \tag{24}$$

$$P_5 \sim |S_{13}|^2|b_3|^2\left|\Gamma - \left(-\frac{1}{S_{13}}\right)\right|^2 \tag{25}$$

$$P_6 \sim \left|\frac{S_{12}}{S_{13}}\right|^2 |b_3|^2. \tag{26}$$

In this case, the three q-values are $-1/S_{13}$, 0, and $-1/S_{13}$. Evidently, the properties of the six-port junction are far from ideal as regards suitability for six-port measurements. Likewise, the cases with the test device connected to ports 2 or 4 lead to the same conclusion.

A circuit with different properties, actually a seven-port, is obtained by connecting a directional coupler with a power detector to the input line in Fig. 3. This detector would then serve as a reference detector, measuring the power incident on the test device. The q-values associated with detectors 1–4 should then ideally be symmetrically distributed around the origin, having equal amplitude, and being spaced 90° apart.

An analysis similar to the one performed above for the case with the test device at port 3 yields the following seven-port q-values:

$$q_1 = -\frac{1}{S_{13}} \tag{27}$$

$$q_2 = -\frac{S_{14}}{S_{12}\cdot S_{13}} \tag{28}$$

$$q_3 = -\frac{1}{S_{13}} \tag{29}$$

$$q_4 = -\frac{S_{12}}{S_{13}\cdot S_{14}}. \tag{30}$$

Inserting the S-parameters derived for the matched, symmetrical six-port junction with equal power distribution, we find that $|q_1|=|q_2|=|q_3|=|q_4|=\sqrt{5}$. However, $\arg(q_3)=\arg(q_1)$ and $\arg(q_4)=\arg(q_2)=\arg(q_1)+\pi$. Consequently, two of the power detectors give information redundant with that obtained from two of the others. The same conclusion results when the test device is connected to port 2 or port 4 of the six-port junction.

Though the cases treated above are not exhaustive regarding ways to use the matched, symmetrical six-port junction in connection with six-port measurements, they give a clear indication of its unsuitability in some simple configurations.

IV. The Equivalent Admittance of the Symmetrical, Reciprocal Six-Port Junction

It was shown above that signal dividing circuits of several different types can be obtained based on the matched, symmetrical, reciprocal six-port junction. An important concept in connection with the matching of the circuit is its equivalent admittance [11], [12]. This quantity has the property that if a two-port network can be found that matches into this admittance, then the same matching network connected in each six-port arm will match the six-port. The problem of matching the six-port is thus reduced to the much simpler problem of matching a one-port, provided that the equivalent admittance is known.

In order to derive an expression for the equivalent admittance, we will follow a procedure similar to the one presented by Riblet [11] for the symmetrical nonreciprocal three-port junction. Consider, therefore, a symmetrical, reciprocal, lossless six-port junction, provided with identical, lossless matching networks at each port. The scattering matrix eigenvalues of the matched symmetrical six-port S_i', $i=1\cdots 4$, must, according to (2), satisfy

$$S_1'+2S_2'+2S_3'+S_4'=0. \tag{31}$$

With the aid of the angles α and β defined in Fig. 1, the arguments θ_2', θ_3', and θ_4' of S_2', S_3', and S_4' can be related to $\theta_1'=\arg(S_1')$ by

$$\theta_2'=\theta_1'-\alpha-\beta \tag{32}$$

$$\theta_3'=\theta_1'-\alpha+\beta-\pi \tag{33}$$

$$\theta_4'=\theta_1'-2\alpha+\pi. \tag{34}$$

Observing that the eigensusceptances of the matched six-port junction, Y_i', $i=1\cdots 4$, are related to θ_i' by

$$Y_i'=\tan\left(-\frac{\theta_i'}{2}\right) \tag{35}$$

we find that (32)–(34) imply the following relations among the Y_i''s:

$$Y_2'=\frac{k_1+k_2Y_1'}{k_2-k_1Y_1'} \tag{36}$$

$$Y_3'=\frac{k_3Y_1'-k_4}{k_3+k_4Y_1'} \tag{37}$$

$$Y_4'=\frac{k_5Y_1'-k_6}{k_5+k_6Y_1'} \tag{38}$$

where $k_1=x+y$, $k_2=1-xy$, $k_3=x-y$, $k_4=1+xy$, $k_5=2x$, $k_6=1-x^2$, and where $x=\tan(\alpha/2)$ and $y=\tan(\beta/2)$.

A comparison with the result derived by Riblet in his analysis of symmetrical four-port circulators and hybrids, which also have four independent eigenvalues and where also three conditions on the Y''s were derived [13], seems to indicate that the eigensusceptances would have to be related in a specific way to enable matching of the six-port. However, this is not the case, as will be shown below.

The eigensusceptances after matching as derived above are related to those before matching Y_i through

$$Y_i'=\frac{C+DY_i}{A-BY_i}, \qquad i=1\cdots 4 \tag{39}$$

where A, jB, jC, and D are the elements of the transfer matrix of the matching network. Choosing a reference plane before matching so that $Y_1=\infty$ and denoting admittances at this reference plane by * quantities, we get, upon insertion of (39) into (36)–(38) after simplification, and using the fact that $AD+BC=1$ for reciprocal matching networks

$$k_1(B^2+D^2)Y_2^*=k_1(AB-CD)-k_2 \tag{40}$$

$$k_4(B^2+D^2)Y_3^*=k_4(AB-CD)+k_3 \tag{41}$$

$$k_6(B^2+D^2)Y_4^*=k_6(AB-CD)+k_5. \tag{42}$$

From (40) and (41) we derive

$$(B^2+D^2)\frac{k_1k_4(Y_3^*-Y_2^*)}{k_1k_3+k_2K_4}=1 \tag{43}$$

$$(B^2+D^2)\frac{k_1k_3Y_2^*+k_2k_4Y_3^*}{k_1k_3+k_2k_4}=AB-CD \tag{44}$$

from which we identify the equivalent admittance of the six-port at the chosen reference plane Y_{eq}^* as being [11]

$$Y_{eq}^*=\frac{k_1k_4(Y_3^*-Y_2^*)+j(k_1k_3Y_2^*+k_2k_4Y_3^*)}{k_1k_3+k_2k_4}. \tag{45}$$

Similarly, by combining (40) and (42), we get

$$Y_{eq}^*=\frac{k_1k_6(Y_4^*-Y_2^*)+j(k_1k_5Y_2^*+k_2k_6Y_4^*)}{k_1k_5+k_2k_6}. \tag{46}$$

The two expressions for Y_{eq}^* in (45) and (46) must be equal, since they are valid for the same circuit. Combining (45) and (46), we find that the following condition must be satisfied:

$$k_1(k_4k_5-k_3k_6)Y_2^*-k_4(k_1k_5+k_2k_6)Y_3^* +k_6(k_2k_4+k_1k_3)\cdot Y_4^*=0. \tag{47}$$

This condition and the expression for the equivalent admittance (45) can be readily generalized to an arbitrary reference plane by using the method outlined in [11] and [13]. The result is

$$Y_{eq}=\frac{(1+Y_1^2)G^*+j\left[G^{*2}Y_1-(1-Y^*Y_1)(Y^*+Y_1)\right]}{G^{*2}+(Y^*+Y_1)^2} \tag{48}$$

where

$$G^* = \frac{k_1 k_4}{k_1 k_3 + k_2 k_4}\left[\frac{Y_3 + \frac{1}{Y_1}}{1 - \frac{Y_3}{Y_1}} - \frac{Y_2 + \frac{1}{Y_1}}{1 - \frac{Y_2}{Y_1}}\right] \tag{49}$$

$$Y^* = \frac{1}{k_1 k_3 + k_2 k_4}\left[k_1 k_3 \frac{Y_2 + \frac{1}{Y_1}}{1 - \frac{Y_2}{Y_1}} + k_2 k_4 \frac{Y_3 + \frac{1}{Y_1}}{1 - \frac{Y_3}{Y_1}}\right] \tag{50}$$

and

$$k_1(k_4 k_5 - k_3 k_6)\cdot\frac{Y_2 + \frac{1}{Y_1}}{1 - \frac{Y_2}{Y_1}} - k_4(k_1 k_5 + k_2 k_6)\cdot\frac{Y_3 + \frac{1}{Y_1}}{1 - \frac{Y_3}{Y_1}} + k_6(k_2 k_4 + k_1 k_3)\cdot\frac{Y_4 + \frac{1}{Y_1}}{1 - \frac{Y_4}{Y_1}} = 0. \tag{51}$$

By regrouping the terms, we find that (51), after simplification, can be rewritten

$$k_1 k_4 k_5 \frac{(Y_1^2+1)(Y_2 - Y_3)}{(Y_1 - Y_2)(Y_1 - Y_3)} + k_1 k_3 k_6 \frac{(Y_1^2+1)(Y_4 - Y_2)}{(Y_1 - Y_4)(Y_1 - Y_2)} + k_2 k_4 k_6 \frac{(Y_1^2+1)(Y_4 - Y_3)}{(Y_1 - Y_4)(Y_1 - Y_3)} = 0. \tag{52}$$

Observing that the common factor Y_1^2+1 is always positive, it can be removed and we get

$$k_1 k_4 k_5 (Y_1 - Y_4)(Y_2 - Y_3) - k_1 k_3 k_6 (Y_1 - Y_3)(Y_2 - Y_4) - k_2 k_4 k_6 (Y_1 - Y_2)(Y_3 - Y_4) = 0. \tag{53}$$

Recalling the definitions of $k_1 \cdots k_6$ given above, we find that they are functions of the angles α and β. Equation (53) gives one condition on the choice of these angles. In order to determine them, we need, however, one more condition. Such a condition can be derived from the geometric configuration of the four scattering matrix eigenvalues (see Fig. 1). Thus, we find that

$$\sin\alpha = 2\sin\beta. \tag{54}$$

By the use of the proper trigonometric identities, it can be shown that (54) implies that

$$y = \frac{1}{x}\left(1 + x^2 - \sqrt{1 + x^2 + x^4}\right) \tag{55}$$

where x and y are simply related to $k_1 \cdots k_6$ as defined above.

A determination of the equivalent admittance for a symmetrical, reciprocal six-port junction would thus include the following:

1) the determination of $Y_1 \cdots Y_4$ by analysis of the given junction,
2) the choice of x in (55) so that the condition in (53) is satisfied, thereby defining $k_1 \cdots k_6$,
3) the calculation of the equivalent admittance from (48), (49), and (50) using these values of $k_1 \cdots k_6$.

It should be observed that the relations (32)–(34) between the scattering matrix eigenvalue arguments imply a choice of one of two complementary configurations as exemplified in Fig. 1. The solution of Y_{eq} for the other configuration gives identical results except for a difference in the sign of $\mathrm{Re}\{Y_{eq}\}$. The determination of Y_{eq} should, therefore, be accompanied by checking for the proper sign of $\mathrm{Re}\{Y_{eq}\}$ in (48).

V. The Design of a Stripline Five-Way Power Divider

To exemplify the design of a component based on the symmetrical, reciprocal six-port junction, it was decided to attempt to build a matched stripline five-way equal power divider with a center frequency of 10 GHz. Based on the analysis by Davies and Cohen of symmetrical stripline junctions [14], it is found that the eigenreactances of a symmetrical, reciprocal six-port junction can be expressed as

$$Z_I = -\left[\frac{6\psi Z_s}{\pi}\sum_{n=6p+I}\left[\frac{\sin(n\psi)}{n\psi}\right]^2\frac{J_n(X)}{J_n'(X)}\right],$$
$$I = 1\cdots 4, \quad p = 0, \pm 1, \pm 2, \cdots \tag{56}$$

where ψ is half the coupling angle of the connecting striplines, Z_s is the normalized characteristic impedance of the connecting striplines, $J_n(X)$ is the Bessel function of order n, and where

$$X = \omega\sqrt{\mu\epsilon}\,R$$

R being the junction radius. The equivalent admittance can then be calculated according to the theory given above. For the special case of equal power division $\alpha = \tan^{-1}(2)$ and the condition (53) can be simplified to

$$6(Y_1 - Y_4)(Y_2 - Y_3) - (Y_1 - Y_3)(Y_2 - Y_4) - 3(Y_1 - Y_2)(Y_3 - Y_4) = 0. \tag{57}$$

It was assumed that the circuit would be built using a laminate with a thickness corresponding to a ground-plane spacing of 3.15 mm and dielectric constant $\epsilon_r = 2.33$. The problem is then to find a set of variables that can be adjusted to satisfy the design goal. Further, it is desirable that each design variable be associated if possible with essentially only one design objective in order to facilitate a rapid optimization.

To obtain a basis for a simple design routine, we will examine the conditions (53) and (55) on the angles α and β in combination with the expression for the reactances (56). Retaining only the lowest order terms in the series expansion of (56), we get for the eigensusceptances

$$Y_I \simeq -\frac{\pi}{6\psi Z_s}\left[\frac{n\psi}{\sin(n\psi)}\right]^2\frac{J_n'(X)}{J_n(X)} \tag{58}$$

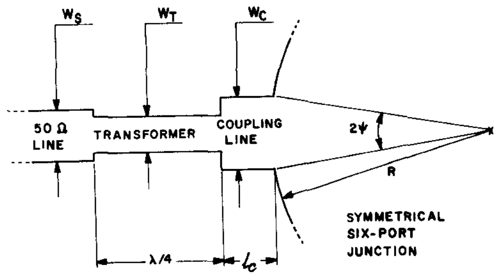

Fig. 4. A matching network for the symmetrical six-port junction.

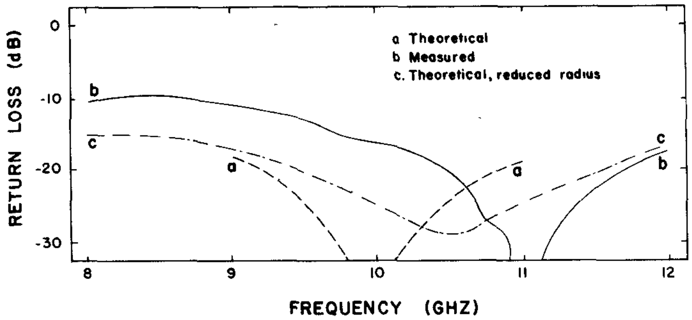

Fig. 5. Return loss of a five-way power divider based on the symmetrical, reciprocal six-port junction. (a) Calculated according to initial design (---). (b) Measured (——). (c) Calculated for a junction with slightly smaller effective radius than in (a) (-·-).

where $I=1\cdots 4$ and $n=I-1$. Further, for small angles ψ, the approximation $\sin(n\psi)\simeq n\psi$ is valid, and (58) can be simplified to

$$Y_I \simeq -\frac{\pi}{6\psi Z_s}\cdot\frac{J_n'(X)}{J_n(X)}. \tag{59}$$

Upon insertion of (59) into (53), we get after simplification

$$k_1k_4k_5\left[\frac{J_0'(X)}{J_0(X)}-\frac{J_3'(X)}{J_3(X)}\right]\left[\frac{J_1'(X)}{J_1(X)}-\frac{J_2'(X)}{J_2(X)}\right] - k_1k_3k_6\left[\frac{J_0'(X)}{J_0(X)}-\frac{J_2'(X)}{J_2(X)}\right]\left[\frac{J_1'(X)}{J_1(X)}-\frac{J_3'(X)}{J_3(X)}\right] - k_2k_4k_6\left[\frac{J_0'(X)}{J_0(X)}-\frac{J_1'(X)}{J_1(X)}\right]\left[\frac{J_2'(X)}{J_2(X)}-\frac{J_3'(X)}{J_3(X)}\right]\simeq 0. \tag{60}$$

Equation (60) has the interesting feature that for any given power division, corresponding to a specified set of $k_1\cdots k_6$, the solution is dependent only on $X=\omega\sqrt{\mu\epsilon}\,R$. If, as in the present design problem, the operating frequency and the material parameters μ and ϵ are given, then the desired power division can be obtained by the proper adjustment of R. When R is determined, the coupling angle ψ can be changed virtually without disturbing the power division. The solution of (60) for the case with equal power division is $X=2.875$, reasonably close to the multimode solution obtained below.

A configuration which permits a simple synthesis is shown in Fig. 4. The circuit consists of a symmetrical junction of radius R to the ports of which are connected a short coupling line and transformer a quarter wavelength long at the chosen center frequency. The following synthesis routine was used.

1) Choose ψ (which defines W_c).
2) Find R so that (57) is satisfied at 10 GHz to assure equal power division when the circuit is matched.
3) Find l_c so that $\mathrm{Im}\{Y_{eq}\}=0$ at 10 GHz, the coupling lines included.
4) Choose W_T to make the characteristic admittance of the transformers $Y_T=\sqrt{\mathrm{Re}\{Y_{eq}\}}$ at 10 GHz.

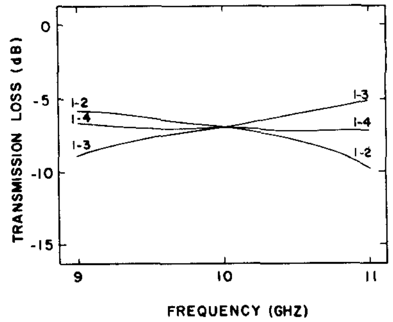

Fig. 6. Theoretical transmission loss of the power divider of Fig. 5(a).

Designs were performed corresponding to various values of the coupling angle ψ. Analysis of the circuits indicated at 20-dB reflection bandwidth of about 15 percent increasing somewhat for small values of ψ. The bandwidth corresponding to a deviation of the coupling to the output ports of at most 1 dB from the ideal value of -6.98 dB is, however, only about 10 percent and is approximately constant when varying ψ.

Since the bandwidth of the power divider is nearly independent of ψ, a value of ψ was chosen so as to make $\mathrm{Re}\{Y_{eq}\}=(50\ \Omega)^{-1}$ at 10 GHz, thereby eliminating the need for the transformers, and consequently minimizing the size of the circuit. The theoretical performance of the circuit is shown in Figs. 5(a) and 6. The design data are $R=9.90$ mm, $\psi=15.2°$, $l_c=1.95$ mm, $W_c=5.19$ mm, and $W_t=W_s=3.89$ mm. The value of $X=\omega\sqrt{\mu\epsilon}\,R$ for this circuit at the frequency for equal power division is 3.166.

An experimental five-way power divider was fabricated using this design data and is pictured in Fig. 7. The whole circuit pattern was reduced a small amount in size to compensate for the fringing fields around the edge [15]. The design data then became $R=9.22$ mm, $l_c=1.85$ mm, $W_c=3.81$ mm, $W_t=W_s=2.49$ mm. The experimental performance is shown in Figs. 5(b) and 8. Theoretical and experimental curves correspond well. However, the performance of the experimental circuit is centered at, or slightly below, 11 GHz instead of the theoretically predicted 10 GHz. Assuming that this discrepancy might be due to smaller than expected fringing fields around the junction, a new analysis was performed using $R=9.22$ mm, the radius of the physical resonator (see Figs. 5(c) and 9). The theoretical and experimental performances are then very similar. Fine-tuning of the circuit to adjust the center frequency and correct for any deviations from ideal power division was considered to be outside the scope of this paper.

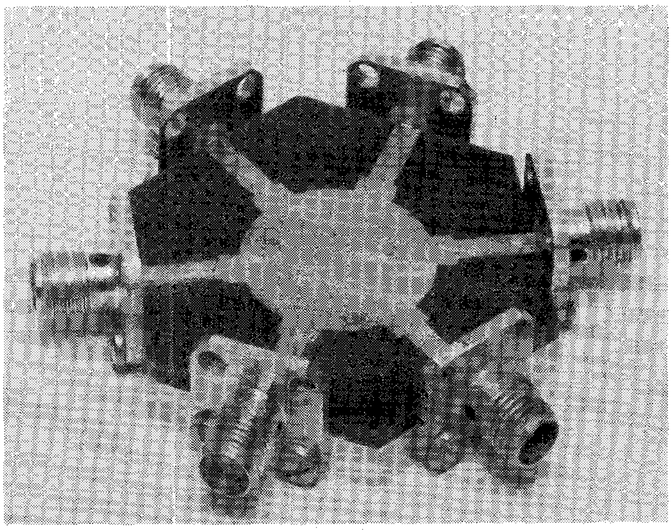

Fig. 7. Experimental one-to-five power divider.

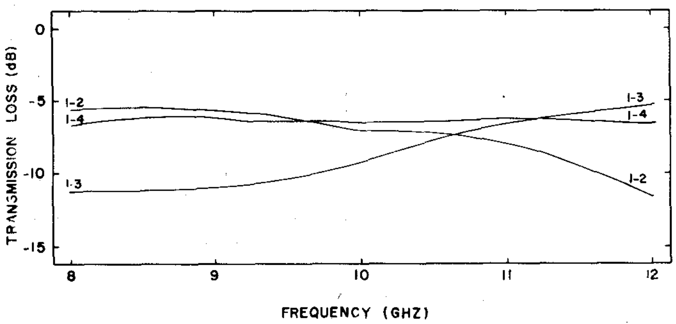

Fig. 8. Measured transmission loss of an experimental stripline power divider.

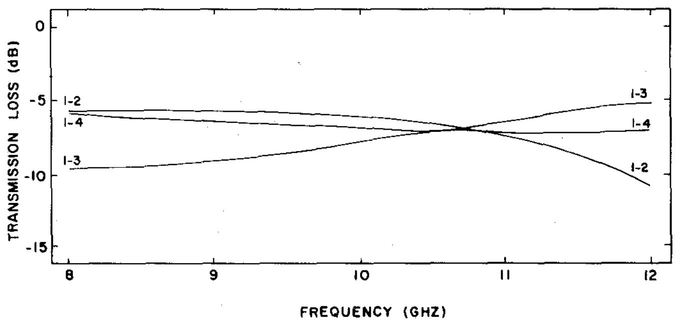

Fig. 9. Theoretical transmission loss of the power divider according to Fig. 5(c).

VI. Conclusions

The scattering matrix element-eigenvalue relations for a reciprocal, symmetrical six-port junction have been derived. Based on these relations, several possible matched circuits were determined. One of the circuits that can be realized is the five-way equal power divider. An important result is that matching alone is not enough to ensure a unique signal distribution.

The use of the matched, symmetrical, reciprocal six-port junction in connection with six-port measurements was discussed. It was shown that this junction is not suited for such measurements, at least not in the most obvious configurations treated in this paper.

The equivalent admittance of the symmetrical, reciprocal six-port junction, an essential design tool, was derived. The theory was applied to the design of a five-way power divider in stripline. A configuration was found which enables a simple synthesis routine to be used. The measured performance of an experimental power divider agreed well with the theoretical performance.

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Integrated Tunable Cavity Gunn Oscillator for 60-GHz Operation in Image Line Waveguide

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Abstract—**The design, construction, and experimental test results of a mechanically tunable Gunn oscillator using a recessed diode metal coaxial cavity coupled to an image line waveguide is described. The oscillator frequency was changed by about 10-percent by varying the bias post length into the coaxial structure. The oscillator is designed so that both the Gunn diode and resonant cavity can be quickly replaced to provide extended frequency coverage and efficiency. This Gunn diode oscillator has provided up to 15-mW CW power at 60 GHz with 10-percent tuning range.**

I. Introduction

METAL WAVEGUIDE cavity oscillators are available now at 60 GHz. However, there is an increasing requirement for sources for image line millimeter-wave integrated circuits. Up to the present time, very little work has been reported on oscillators compatible with image line technology. A recent development of the Gunn oscillator using quartz image line was reported by Y. W. Chang [1]. In the following, a development is reported which shows how an oscillator can be integrated into image line subsystems with useable power output and good mechanical tuneability.

The millimeter-wave oscillator is designed around a recessed coaxial air-filled metal cavity which is coupled to a dielectric image line. The physical design incorporates a replacable oscillator coaxial cavity and a structure with a replaceable Gunn diode, as shown in Fig. 1.

Manuscript received May 23, 1983; revised August 3, 1983.
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II. Oscillator Physical Design

The circuit as shown in Fig. 1 consists of a brass body 1.5 in long by 0.75 in wide and 0.5 in thick. The Gunn diode is threaded into a smooth brass cylinder which is inserted in the bottom of the brass body to form a resonant cavity. A brass post (0.025-in diameter) is threaded through the tuning top disk mounted on the top of the image line and through a 0.050-in-diameter hole in the dielectric. This provides a means of coupling up from the metal cavity into the image line waveguide. The dc bias voltage is applied to the top tuning disk through the tuning rod to the Gunn diode as shown in Fig. 2. Through the use of this tuning arrangement, the cavity height is variable (as tested) from 0.015 to 0.100 in over which a wide tuning range can be realized. The oscillator resonant (metal) cavity is shown in Fig. 2. The output is coupled through a narrowed opening at the top of a metal cavity into a hole of 0.050-in diameter in the alumina material which forms the image guide structure. A metal disk (0.120-in diameter) cemented to the top of the dielectric (alumina) serves as a bias connection, tuning screw mounted for the bias post, and prevents extraneous radiation from the dielectric by providing a top wall for the metal cavity. The alumina guide is 0.120 in wide, 0.040 in thick, and 1.0 in long. The wave is coupled ideally into the alumina guide in the form of the $E_{11}y$ mode. The alumina end is tapered for impedance matching into a metal waveguide structure for test and evaluation.

Although not shown on Fig. 1, a 100-pF chip capacitor was mounted between the top tuning disk and ground.